Interaction of a nonlinear spin wave and magnetic soliton in a uniaxial anisotropic ferromagnet

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We study the interaction of a nonlinear spin-wave and magnetic soliton in a uniaxial anisotropic ferromagnet. By means of a reasonable assumption and a straightforward Darboux transformation one- and two-soliton solutions in a nonlinear spin-wave background are obtained analytically, and their properties are discussed in detail. In the background of a nonlinear spin wave the amplitude of the envelope soliton has the spatial and temporal period, and soliton can be trapped only in space. The amplitude and wave number of spin wave have the different contribution to the width, velocity, and the amplitude of soliton solutions, respectively. The envelope of solution hold the shape of soliton, and the amplitude of each envelope soliton keeps invariability before and after collision which shows the elastic collision of two envelope soliton in the background of a nonlinear spin wave.

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I. INTRODUCTION

Solitons are the natural results of the nonlinear equation in which the dispersion is compensated by nonlinear effects. With this compensation solitons can travel over long distances with neither attenuation nor change of shape which has the prominent application in high-rate telecommunications with optical fibers in the future. For this reason the study of solitons has received considerable attentions in many fields, such as particle physics, molecular biology, nonlinear optics [1] and condensate physics [2–4]. One of the topics in condensate physics is the nonlinear dynamics of local magnetization in Heisenberg spin chain model which successfully explains the existence of ferromagnetism and antiferromagnetism at temperatures below the Curie temperature. Taking into account the spin-spin interactions, the nonlinear excitations, such as spin wave and magnetic solitons [5, 6], are general phenomena

in ordered ferromagnetic materials. By means of the neutron inelastic scattering [7] and electron spin resonance [8], the magnetic soliton had already been probed experimentally in quasi-one-dimensional magnetic systems. The magnetic soliton, which describes localized magnetization, is an important excitation in the classical Heisenberg spin chain. In particular, the continuum limit for the nonlinear dynamics of magnetization in the classical ferromagnet is governed by the Landau-Lifshitz (L-L) equation [9]. This equation governs a classical nonlinear dynamically system with novel properties. In a one-dimensional case, some types of L-L equation is complete integrable. The isotropic case has been studied in various aspects [10–12], and the construction of soliton solutions of anisotropic L-L equation is also discussed [13–18]. In these studies a variety of techniques, such as inverse scattering transformation [12–15], Riemann-Hilbert approach [16], and Darboux transformation [17, 18], have been applied to construct the soliton solution of L-L equation. Recently, this Heisenberg model has new application in some fields of condensate physics. In magnetic multilayers the magnetization switching and reversal [19], domain-wall dynamics [20] and magnetic solitons [21, 22] were discussed analytically in terms of L-L equation with spin-torque. Taking into account the light-induced and the magnetic dipole-dipole interactions the ferromagnetic state [23] are obtained and the nonlinear excitations, such as spin wave [24] and magnetic solitons [25], has been reported in spinor Bose-Einstein condensates in deep optical lattice.

In this paper, we will study the interaction of a nonlinear spin wave and magnetic solitons in a uniaxial anisotropic ferromagnet. In terms of a reasonable assumption we transform the Landau-Lifshitz equation into an equation of the nonlinear type. By means of a straightforward Darboux transformation we report one- and two-soliton solutions in a nonlinear spin-wave background analytically and discuss their properties in detail.

II. MODEL

In the classical limit, the dynamics of magnetization of a ferromagnet as a function of space and time $\boldsymbol{M}(x,t)$ is determined by the Landau-Lifshitz equation [9]

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{2\mu_0}{\hbar}\mathbf{M}\times\mathbf{H}_{eff}, \tag{1}$$

where μ_0 is Bohr magneton, the effective magnetic field $\mathbf{H}_{eff}$ is equal to the variational derivative of the magnetic crystal energy with respect to the vector $\mathbf{M}$,

$$\mathbf{H}_{eff} = -\frac{\delta E}{\delta \mathbf{M}}, \tag{2}$$

where the magnetic crystal energy function including the exchange energy, anisotropic energy and the Zeeman energy can be written as [5]

$$E = \frac{1}{2} \int \left[A \left(\frac{\partial \mathbf{M}}{\partial x} \right)^2 - \beta M_z^2 - \mathbf{M} \cdot \mathbf{B} \right] d^3x, \tag{3}$$

where A is the exchange constant and β is the uniaxial anisotropic constant, $\beta > 0$ corresponds to easy-axis anisotropy while $\beta < 0$ corresponds to easy-plane type. Substituting Eqs. (2) and (3) into Eq. (1), we get

$$\frac{\hbar}{2\mu_0} \frac{\partial \mathbf{M}}{\partial t} = -A\mathbf{M} \times \frac{\partial^2 \mathbf{M}}{\partial x^2} - \beta \mathbf{M} \times \mathbf{e}_3 (\mathbf{M} \cdot \mathbf{e}_3) - \mu_0 \mathbf{M} \times \mathbf{B}, \tag{4}$$

where $\mathbf{e}_3$ is the unit vector along the z-axis, and $\mathbf{B} = (0, 0, B)$. When the magnetic field is high enough, the deviation of magnetization from the the direction of the field is small. With the consideration that at temperatures well below the Curie temperature the magnitude of magnetization is a constant, i.e., the integral of motion $\mathbf{M}^2 \equiv M_0^2 = constant$, here M_0 is the saturation magnetization, we can introduce a single function Ψ, instead of two independent components of $\mathbf{M}$,

$$\Psi = m_x + i m_y, \;\; m_z = \sqrt{1 - |\Psi|^2}, \tag{5}$$

where $\mathbf{m} \equiv (m_x, m_y, m_z) = \mathbf{M}/M_0$. Thus Eq. (4) becomes

$$\frac{i\hbar}{2\mu_0} \frac{\partial \Psi}{\partial t} - A M_0 \left(m_z \frac{\partial^2}{\partial x^2} \Psi - \Psi \frac{\partial^2}{\partial x^2} m_z \right) = - \left(\beta M_0 m_z + B \right) \Psi. \tag{6}$$

In the ground state, vector $\mathbf{M}$ directs along the anisotropy axis $\mathbf{e}_3$. Now we consider the small deviations of magnetization from the equilibrium direction which corresponding to $m_x^2 + m_y^2 \ll m_z^2$, i.e., $|\psi|^2 \ll 1$, then $m_z \approx 1 - 1/2 \, |\Psi|^2$. In the long-wavelength approximation and the case $\beta > 0$, Eq. (6) may be simplified by keeping only the nonlinear terms of the order of the magnitude of $|\psi|^2 \psi$. As a result, we have the following dimensionless Schrödinger equation:

$$i \frac{\partial \Psi}{\partial t} - \frac{1}{2} \frac{\partial^2 \Psi}{\partial x^2} - |\Psi|^2 \Psi + 2 (1 + \frac{B}{\beta M_0}) \Psi = 0. \tag{7}$$

For convenience we have rescaled space x and time t by $2l_0$ and $1/\omega_0$, where $l_0 = \sqrt{A/\beta}$ is the characteristic magnetic length and $\omega_0 = \beta\mu_0 M_0/\hbar$ is the homogeneous ferromagnetic resonance frequency. The spin wave interaction in a ferromagnet with easy-axis anisotropy is attractive in nature. This attraction leads to the macroscopic phenomena which are associated with the appearance of spatially localized magnetic excitations, i.e., magnetic solitons which is admitted in nonlinear equation (7). So it is very interesting to investigate the dynamics of magnetic soliton in a nonlinear spin wave background which hasn't been well explored. To this purpose we have to get soliton solutions of (7) embedded in a nonlinear spin-wave background. It should be noted that Darboux transformation [26–28] has been developed to construct such solutions. The main idea of this method is that it firstly transforms the nonlinear equation into the Lax representation, and then in terms of a series of reasonable transformations the soliton solutions can be constructed algebraically with an obvious seed solution of the nonlinear equation. By employing Ablowitz-Kaup-Newell-Segur technique we can construct Lax representation for Eq. (7) as follows

$$\begin{aligned} \frac{\partial}{\partial x}\psi &= U\psi, \\ \frac{\partial}{\partial t}\psi &= V\psi, \end{aligned} \tag{8}$$

where $\psi = \begin{pmatrix} \psi_1 & \psi_2 \end{pmatrix}^T$, the superscript "$T$" denotes the matrix transpose. The lax pairs U and V are given in the forms

$$\begin{aligned} U &= \lambda J + P, \\ V &= \left(-i\lambda^2 + \alpha_2\right) J - i\lambda P + \frac{1}{2} i(P^2 + \frac{\partial}{\partial x} P)J, \end{aligned} \tag{9}$$

with

$$J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; P = \begin{pmatrix} 0 & \Psi \\ -\overline{\Psi} & 0 \end{pmatrix}, \alpha_2 = i(1 + \frac{B}{\beta M_0}),$$

where the overbar denotes the complex conjugate. With the natural condition of Eq. (8) $\frac{\partial^2}{\partial x \partial t}\psi = \frac{\partial^2}{\partial t \partial x}\psi$, i.e., $\frac{\partial}{\partial t}U - \frac{\partial}{\partial x}V + [U, V] = 0$, the Eq. (7) can be recovered. Based on the Eq. (8), we can obtain the general one- and two-soliton solution by using a straightforward Darboux transformation [26–28].

III. DARBOUX TRANSFORMATION

In order to clear the correction of soliton solutions we briefly introduce the procedure of getting soliton solutions for the developed Darboux transformation. We consider the following transformation

$$\psi\left[1\right]=\left(\lambda I-S\right)\psi, \tag{10}$$

where $S=K\Lambda K^{-1}$, Λ =diag(λ_1,λ_2) , and K is a nonsingular matrix which satisfies

$$K_x=JK\Lambda+PK. \tag{11}$$

Letting $\psi\left[1\right]$ satisfy the Lax equation

$$\frac{\partial}{\partial x}\psi\left[1\right]=U_1\psi\left[1\right], \tag{12}$$

where $U_1=\lambda J+P_1$, $P_1=\begin{pmatrix}0 & \Psi_1\\ -\overline{\Psi}_1 & 0\end{pmatrix}$, and with the help of Eqs. (9), (10) and (11), we obtain the Darboux transformation for Eq. (7) from Eq. (12) in the form

$$\Psi_1=\Psi+2S_{12}. \tag{13}$$

The above equation implies that a new solution of Eq. (7) can be obtained if S is known. To obtain the expression of S it is is easy to verify that, if $\psi=\begin{pmatrix}\psi_1 & \psi_2\end{pmatrix}^T$ is a eigenfunction of Eq. (8) corresponding to the eigenvalue $\lambda=\lambda_1$, then$\begin{pmatrix}-\overline{\psi}_2 & \overline{\psi}_1\end{pmatrix}^T$ is also the eigenfunction, while with the eigenvalue $-\overline{\lambda}_1$. Therefore, K and Λ can be taken the form

$$K=\begin{pmatrix}\psi_1 & -\overline{\psi}_2\\ \psi_2 & \overline{\psi}_1\end{pmatrix},\Lambda=\begin{pmatrix}\lambda_1 & 0\\ 0 & -\overline{\lambda}_1\end{pmatrix}, \tag{14}$$

which ensures that Eq. (11) is held, we can obtain

$$S_{sl}=-\overline{\lambda}_1\delta_{sl}+\left(\lambda_1+\overline{\lambda}_1\right)\frac{\psi_s\overline{\psi}_l}{\psi^T\overline{\psi}},\ \ s,l=1,2, \tag{15}$$

where $\psi^T\overline{\psi}=\left|\psi_1\right|^2+\left|\psi_2\right|^2$. Then Eq. (13) becomes

$$\Psi_1=\Psi+2\left(\lambda_1+\overline{\lambda}_1\right)\frac{\psi_1\overline{\psi}_2}{\psi^T\overline{\psi}}, \tag{16}$$

where $\psi=\begin{pmatrix}\psi_1 & \psi_2\end{pmatrix}^T$ to be determined is the eigenfunction of Eq. (8) corresponding to the eigenvalue λ_1 for the solution Ψ. Thus if Eq. (8) is solved we can generate a new solution

Ψ_1, therefore the new solutions of $\mathbf{m}$ from Eq. (5), of the Eq. (7) from a known solution Ψ which is usually called "seed" solution.

To obtain exact N-order solution of Eq. (7), we firstly rewrite the Darboux transformation in Eq. (16) as in the form

$$\Psi_1 = \Psi + 2\left(\lambda_1 + \overline{\lambda}_1\right) \frac{\psi_1\left[1,\lambda_1\right]\overline{\psi}_2\left[1,\lambda_1\right]}{\psi\left[1,\lambda_1\right]^T\overline{\psi}\left[1,\lambda_1\right]}, \tag{17}$$

where $\psi\left[1,\lambda\right] = \left(\psi_1\left[1,\lambda\right], \psi_2\left[1,\lambda\right]\right)^T$ denotes the eigenfunction of Eq. (9) corresponding to eigenvalue λ. Then repeating above the procedure N times, we can obtain the exact N-order solution

$$\Psi_N = \Psi + 2\sum_{n=1}^{N}(\lambda_n + \overline{\lambda}_n)\frac{\psi_1[n,\lambda_n]\overline{\psi}_2[n,\lambda_n]}{\psi[n,\lambda_n]^T\overline{\psi}[n,\lambda_n]}, \tag{18}$$

where

$$\psi\left[n,\lambda\right] = \left(\lambda - S\left[n-1\right]\right)\cdots\left(\lambda - S\left[1\right]\right)\psi\left[1,\lambda\right],$$
$$S_{sl}\left[j\right] = -\overline{\lambda}_j\delta_{sl} + \left(\lambda_j + \overline{\lambda}_j\right)\frac{\psi_s\left[j,\lambda_j\right]\overline{\psi}_l\left[j,\lambda_j\right]}{\psi\left[j,\lambda_j\right]^T\overline{\psi}\left[j,\lambda_j\right]},$$

here $\psi\left[j,\lambda\right]$ is the eigenfunction corresponding to λ_j for Ψ_{j-1} with $\Psi_0 \equiv \Psi$ and $s, l = 1, 2$, $j = 1, 2, \cdots, n-1$, $n = 2, 3, \cdots, N$. Thus if choosing a "seed" as the basic initial solution, by solving linear characteristic equation system (8), one can construct a set of new solutions of $\mathbf{m}$ from Eqs. (5), (7) and (18).

In order to get soliton solutions in a nonlinear spin wave background we take the initial "seed" solution as

$$\Psi_c = A_c e^{-i(k_c x - \omega_c t)}, \tag{19}$$

which corresponding to a nonlinear spin wave solution of $\mathbf{m}$ as

$$\begin{aligned} m_x &= A_c \cos\left(k_c x - \omega_c t\right), \\ m_y &= -A_c \sin\left(k_c x - \omega_c t\right), \\ m_z &= \sqrt{1 - A_c^2}, \end{aligned} \tag{20}$$

which satisfies the nonlinear dispersion relation $\omega_c = k_c^2/2 - A_c^2 + 2B/\left(\beta M_0\right) + 2$, where the amplitude A_c is small real constants and k_c is the wave number of nonlinear spin wave. Substituting (19) into (7) and solving the linear equation system (8), after the tedious

calculation we have the eigenfunction of Eq. (8) corresponding to eigenvalue λ in the form

$$\begin{aligned} \psi_1 &= LC_1 e^{\Theta_1} + A_c C_2 e^{\Theta_2}, \\ \psi_2 &= A_c C_1 e^{-\Theta_2} + LC_2 e^{-\Theta_1}, \end{aligned} \tag{21}$$

where the parameters C_1 and C_2 are the arbitrary complex constants, and the other parameters are defined by

$$\begin{aligned} \Theta_1 &= -\frac{1}{2} i \left(k_c x - \omega_c t\right) + D\left(x + \delta t\right), \\ \Theta_2 &= -\frac{1}{2} i \left(k_c x - \omega_c t\right) - D\left(x + \delta t\right), \\ L &= -\frac{1}{2} i k_c - D - \lambda, \\ D &= \frac{1}{2}\sqrt{\left(i k_c + 2\lambda\right)^2 - 4A_c^2}, \\ \delta &= -i\lambda - \frac{1}{2} k_c, \end{aligned}$$

Following the formulas (5), (18), and (21) we can obtain the one- and two-soliton solutions embedded in a nonlinear spin wave background, respectively.

IV. MODULATION OF ONE-SOLITON SOLUTION BY A NONLINEAR SPIN WAVE

Taking the spectral parameter $\lambda \equiv \lambda_1 = \mu_1/2 + i\nu_1/2$, here μ_1 and ν_1 are real number, in Eq. (21) and substituting them into Eq. (17), we obtain the one-soliton solution from Eqs. (5) and (18) as follows

$$\begin{aligned} m_x &= A_c \cos\varphi + \frac{\mu_1}{\Delta_1}\left(Q_2 \sin\varphi + Q_1 \cos\varphi\right), \\ m_y &= -A_c \sin\varphi + \frac{\mu_1}{\Delta_1}\left(Q_2 \cos\varphi - Q_1 \sin\varphi\right), \\ m_z &= \sqrt{1 - \left(A_c + \frac{\mu_1}{\Delta_1} Q_1\right)^2 - \left(\frac{\mu_1}{\Delta_1} Q_2\right)^2}, \end{aligned} \tag{22}$$

where

$$\begin{aligned} \varphi &= k_c x - \omega_c t, \\ \theta_1 &= 2D_{1R} x + 2\left(D_1 \delta_1\right)_R t + 2x_0, \\ \Phi_1 &= 2D_{1I} x + 2\left(D_1 \delta_1\right)_I t - 2\varphi_0, \end{aligned} \tag{23}$$

$$
\begin{aligned}
D_1 &= \sqrt{(ik_c/2+\lambda_1)^2 - A_c^2}, \\
L_1 &= -ik_c/2 - D_1 - \lambda_1, \\
\delta_1 &= -i\lambda_1 - k_c/2, \\
Q_1 &= 2A_c L_{1R}\cosh\theta_1 + \left(|L_1|^2 + A_c^2\right)\cos\Phi_1, \\
Q_2 &= 2A_c L_{1I}\sinh\theta_1 + \left(|L_1|^2 - A_c^2\right)\sin\Phi_1, \\
\Delta_1 &= \left(|L_1|^2 + A_c^2\right)\cosh\theta_1 + 2A_c L_{1R}\cos\Phi_1, \\
x_0 &= -\left(\ln|C_2/C_1|\right)/2,\ \varphi_0 = \left[\arg\left(C_2/C_1\right)\right]/2,
\end{aligned} \tag{24}
$$

here the subscript R and I represent the real part and image part, respectively. The parameters C_1, C_2 are the arbitrary complex constants. It should be noted that A_c, k_c, x_0, ϕ_0, and the complex variable λ_1 are the adjustable parameters whose values characterize the independent solutions.

When the spin-wave amplitude vanishes, namely $A_c = 0$, the solution in Eq. (22) reduces to the solution in the form

$$
\begin{aligned}
m_x &= \frac{\mu_1}{\cosh\theta_1}\cos\left(\Phi_1 - \gamma\right), \\
m_y &= \frac{\mu_1}{\cosh\theta_1}\sin\left(\Phi_1 - \gamma\right), \\
m_z &= \sqrt{1 - \frac{\mu_1^2}{\cosh^2\theta_1}},
\end{aligned} \tag{25}
$$

where

$$
\begin{aligned}
\gamma &= -2\left(1 + \frac{B}{\beta M_0}\right)t, \\
\theta_1 &= \mu_1 x + \mu_1\nu_1 t + 2x_0, \\
\Phi_1 &= \nu_1 x - \frac{1}{2}\left(\mu_1^2 - \nu_1^2\right)t - 2\varphi_0.
\end{aligned}
$$

The parameters $-2x_0/\mu_1$ and $2\varphi_0/\nu_1$ represent the initial center position and initial phase. The expression (25) describes a magnetization precession characterized by the deviations μ_1 from the ground state, the amplitude $\sqrt{1-\mu_1^2}$, the velocity $-\nu_1$, and the frequency $\left(\mu_1^2 - \nu_1^2\right)/2 - 2 - 2B/\left(\beta M_0\right)$ which shows that the magnetic field contribute to precession frequency only. In the other hand, when $\mu_1 = 0$, the solution (22) reduces to a nonlinear

spin-wave solution

$$\begin{aligned} m_x &= A_c \cos\varphi, \\ m_y &= -A_c \sin\varphi, \\ m_z &= \sqrt{1-A_c^2}. \end{aligned} \tag{26}$$

Thus the solution (22) describes a soliton solution of ferromagnet embedded in a nonlinear spin-wave background (20). The properties of envelope soliton is characterized by the width $1/(2D_{1R})$, the wave number of soliton $k_s = 2D_{1I}$, and the envelope velocity $v_1 = -(D_1\delta_1)_R/D_{1R}$. With the expressions of D_1 and δ_1 we found that the amplitude A_c and wave number k_c of spin wave have the different contribution to soliton solutions. The width becomes large with the increasing A_c, however, becomes decreasing with the increasing k_c. In the other hand the velocity of envelope soliton increases continuously with the increasing A_c, however, the increasing at first, then decreasing rapidly with the increasing k_c. From Eq. (23) we can directly see that when $D_{1I}\delta_{1I} = \delta_{1R}D_{1R}$, the parameters θ_1 depends only on x which implies the envelope velocity $-(D_1\delta_1)_R/D_{1R}$ becomes zero, i.e., the magnetic soliton is trapped in space by the nonlinear spin wave which shows a new technique of management for the soliton. It should be noted this condition is determined by the amplitudes μ_1, A_c of soliton and spin-wave, and the wave numbers ν_1, k_c of soliton and spin-wave, respectively. It also can be seen that the amplitude of soliton in m_z is modulated by the spin wave, and characterized by the spatial and temporal period along the line $x = -(D_1\delta_1)_R t/D_{1R} - x_0/D_{1R}$, denoted by $\pi(D_1\delta_1)_R/[\delta_{1I}(D_{1R}^2 + D_{1I}^2)]$ and $\pi D_{1R}/[\delta_{1I}(D_{1R}^2 + D_{1I}^2)]$ obtained from Eqs. (22), (23) and (24), respectively. With the increasing A_c the envelope of solution hold the shape of soliton, and the envelope valley is more deep. Moreover, we can see spin wave move the center position of soliton.

V. TWO-SOLITON INTERACTION MODULATED BY A NONLINEAR SPIN WAVE

It is well known that soliton has the properties of the physical particles in the process of collision. Therefore, to investigate the magnetic soliton collision modulated by a nonlinear spin wave is an very interesting phenomenon in spin dynamics. To approach this we should obtain the two-soliton solution of $\mathbf{m}$ from Eqs. (5), (7) and (18). Taking the spectral

parameter $\lambda = \lambda_j \equiv \mu_j/2 + i\nu_j/2$, $j = 1, 2$, we obtain the two-soliton solution of ferromagnet as

$$\begin{aligned}
m_x &= A_c \cos\varphi + \frac{1}{F_2}\left(G_{2I}\sin\varphi + G_{2R}\cos\varphi\right), \\
m_y &= -A_c \sin\varphi + \frac{1}{F_2}\left(G_{2I}\cos\varphi - G_{2R}\sin\varphi\right), \\
m_z &= \sqrt{1 - \left(A_c + \frac{G_{2R}}{F_2}\right)^2 - \frac{G_{2I}^2}{F_2^2}},
\end{aligned} \tag{27}$$

where the subscript R and I represent the real part and image part, respectively, and the other parameters are defined by

$$\begin{aligned}
\theta_j &= 2D_{jR}x + 2\left(D_j\delta_j\right)_R t + 2x_{0,j}, \\
\Phi_j &= 2D_{jI}x + 2\left(D_j\delta_j\right)_I t - 2\varphi_{0,j}, \\
\varphi &= k_c x - \omega_c t, \\
\Delta_1 &= \left(|L_1|^2 + A_c^2\right)\cosh\theta_1 + 2A_c L_{1R}\cos\Phi_1, \\
\Delta_2 &= \left(|L_2|^2 + A_c^2\right)\cosh\theta_2 + 2A_c L_{2R}\cos\Phi_2, \\
\Delta_3 &= \left(\overline{L}_1 L_2 + A_c^2\right)\left[e^{\frac{1}{2}(\theta_1 - i\Phi_1 + \theta_2 + i\Phi_2)} + e^{-\frac{1}{2}(\theta_1 - i\Phi_1 + \theta_2 + i\Phi_2)}\right] \\
&\quad + A_c\left(\overline{L}_1 + L_2\right)\left[e^{\frac{1}{2}(\theta_1 - i\Phi_1 - \theta_2 - i\Phi_2)} + e^{-\frac{1}{2}(\theta_1 - i\Phi_1 - \theta_2 - i\Phi_2)}\right],
\end{aligned} \tag{28}$$

$$\begin{aligned}
F_2 &= \zeta_1\Delta_1\Delta_2 - \mu_1\mu_2\left|\frac{\Delta_3}{2}\right|^2, \\
G_2 &= \mu_2\zeta_1\Delta_1 f_2 + \mu_1\zeta_1\Delta_2 f_1 - \frac{1}{2}\zeta_2 f_3\Delta_3 - \frac{1}{2}\overline{\zeta}_2\overline{\Delta}_3 f_4, \\
f_1 &= A_c\left(L_1 e^{\theta_1} + \overline{L}_1 e^{-\theta_1}\right) + |L_1|^2 e^{i\Phi_1} + A_c^2 e^{-i\Phi_1}, \\
f_2 &= A_c\left(L_2 e^{\theta_2} + \overline{L}_2 e^{-\theta_2}\right) + |L_2|^2 e^{i\Phi_2} + A_c^2 e^{-i\Phi_2}, \\
f_3 &= [A_c L_1 e^{\frac{1}{2}(\theta_1 + i\Phi_1)} + A_c^2 e^{-\frac{1}{2}(\theta_1 + i\Phi_1)}]e^{\frac{1}{2}(\theta_2 - i\Phi_2)} \\
&\quad + \overline{L}_2[L_1 e^{\frac{1}{2}(\theta_1 + i\Phi_1)} + A_c e^{-\frac{1}{2}(\theta_1 + i\Phi_1)}]e^{-\frac{1}{2}(\theta_2 - i\Phi_2)}, \\
f_4 &= [A_c L_2 e^{\frac{1}{2}(\theta_2 + i\Phi_2)} + A_c^2 e^{-\frac{1}{2}(\theta_2 + i\Phi_2)}]e^{\frac{1}{2}(\theta_1 - i\Phi_1)} \\
&\quad + \overline{L}_1[L_2 e^{\frac{1}{2}(\theta_2 + i\Phi_2)} + A_c e^{-\frac{1}{2}(\theta_2 + i\Phi_2)}]e^{-\frac{1}{2}(\theta_1 - i\Phi_1)},
\end{aligned} \tag{29}$$

$$
\begin{aligned}
\zeta_1 &= \left|\lambda_2+\overline{\lambda}_1\right|^2,\ \zeta_2=\mu_1\mu_2\left(\overline{\lambda}_2+\lambda_1\right),\\
D_j &= \sqrt{\left(ik_c/2+\lambda_j\right)^2-A_c^2},\\
L_j &= -ik_c/2-D_j-\lambda_j,\\
\delta_j &= -i\lambda_j-k_c/2,\\
x_{0,j} &= -\left(\ln\left|C_{2,j}/C_{1,j}\right|\right)/2,\ \varphi_{0,j}=\left[\arg\left(C_{2,j}/C_{1,j}\right)\right]/2,
\end{aligned}
\tag{30}
$$

where $C_{1,j}, C_{2,j}$ are the arbitrary complex constants, $j=1,2$. The subscript R and I represent the real part and image part, respectively. It should be noted that A_c, k_c, $x_{0,j}$, $\phi_{0,j}$, and the complex variable λ_j, $j=1,2$, are the adjustable parameters whose values characterize the independent solutions. In general, the solution (27) represents the interaction of two one-soliton solution modulated by the nonlinear spin-wave (20) of ferromagnet. The properties of each envelope soliton are characterized by the width $1/\left(2D_{jR}\right)$, the wave number of soliton $2D_{jI}$, and the envelope velocity $v_j=-\left(D_j\delta_j\right)_R/D_{jR}$, $j=1,2$. From Eq. (28) we can directly see that when $D_{jI}\delta_{jI}=\delta_{jR}D_{jR}$, $j=1,2$, the parameters θ_j depends only on x which implies the velocity of each envelope becomes zero, i.e., the magnetic soliton is trapped in space by the nonlinear spin wave. It also can be seen that the condition is obviously determined by the amplitudes μ_j, A_c of soliton and spin-wave, and the wave numbers ν_j, k_c of soliton and spin-wave, $j=1,2$, respectively. We also observe that the amplitude of each envelope soliton in m_z is changed by the spin wave, and characterized by the spatial and time period along the line $x=-\left(D_j\delta_j\right)_R t/D_{jR}-x_0/D_{jR}$, denoted by $\pi\left(D_j\delta_j\right)_R/\left[\delta_{jI}\left(D_{jR}^2+D_{jI}^2\right)\right]$ and $\pi D_{jR}/\left[\delta_{jI}\left(D_{jR}^2+D_{jI}^2\right)\right]$, $j=1,2$, for each envelope soliton, respectively. With the action of spin wave the envelope of solution hold the shape of soliton, and the amplitude of each envelope soliton keeps invariability before and after collision which shows the elastic collision of two envelope soliton in the nonlinear spin wave background. Moreover, we can see spin wave move the center position of soliton oppositely before and after collision.

When $\mu_j=0$, $j=1,2$, the solution (27) reduces to the nonlinear wave solution (20). As the nonlinear spin-wave amplitude vanishes, $A_c=0$, the two soliton solution (27) reduces

to

$$
\begin{aligned}
m_x &= \frac{1}{F_2}\left(G_{2I}\sin\gamma + G_{2R}\cos\gamma\right), \\
m_y &= \frac{1}{F_2}\left(G_{2I}\cos\gamma - G_{2R}\sin\gamma\right), \\
m_z &= \sqrt{1-\frac{G_{2R}^2+G_{2I}^2}{F_2^2}},
\end{aligned} \tag{31}
$$

where

$$
\begin{aligned}
\gamma &= -2\left(1+\frac{B}{\beta M_0}\right)t, \\
\theta_j &= \mu_j x + \mu_j \nu_j t + 2x_0, \\
\Phi_j &= \nu_j x - \frac{1}{2}\left(\mu_j^2-\nu_j^2\right)t - 2\varphi_0, \\
F_2 &= \zeta_1\cosh\theta_1\cosh\theta_2 - \frac{1}{2}\mu_1\mu_2\left[\cos(\Phi_1-\Phi_2)+\cosh(\theta_1+\theta_2)\right], \\
G_2 &= \left[\mu_2\zeta_1\cosh\theta_1 - \frac{1}{2}\left(\zeta_2 e^{\theta_1}+\overline{\zeta}_2 e^{-\theta_1}\right)\right]e^{i\Phi_2} \\
&\quad + \left[\mu_1\zeta_1\cosh\theta_2 - \frac{1}{2}\left(\zeta_2 e^{-\theta_2}+\overline{\zeta}_2 e^{\theta_2}\right)\right]e^{i\Phi_1}.
\end{aligned}
$$

The solution (31) describes a general scattering process of two solitary waves with different center velocities $-\nu_1$ and $-\nu_2$, different phases $\Phi_1-\gamma$ and $\Phi_2-\gamma$. Before collision, they move towards each other, one with velocity $-\nu_1$ and shape variation frequency $\Omega_1 = \left(\mu_1^2-\nu_1^2\right)/2-2-2B/\left(\beta M_0\right)$ and the other with $-\nu_2$ and $\Omega_2 = \left(\mu_2^2-\nu_2^2\right)/2-2-2B/\left(\beta M_0\right)$. Asymptotically, the two-soliton waves (31) can be written as a combination of two one-soliton solutions (25) with different amplitudes and phases. The asymptotic form of two-soliton solution in limits $t\to-\infty$ and $t\to\infty$ is similar to that of the one-soliton solution (25). During collision there is no amplitude exchange among three components m_x, m_y and m_z, however, a phase and the center position change for each magnetization vector soliton. This interaction between two magnetic solitons is called elastic collision.

VI. CONCLUSION

In conclusion, we study the dynamics of the magnetic soliton modulated by a nonlinear spin-wave in a uniaxial anisotropic ferromagnet. In terms of a reasonable assumption we transform the Landau-Lifshitz equation into an equation of the nonlinear type. By means of

a straightforward Darboux transformation one- and two-soliton solutions in nonlinear spin-wave background are obtained analytically and their properties are discussed in detail. Our results show that in the background of a nonlinear spin wave the amplitude of the envelope soliton has the spatial and temporal period. The soliton can be trapped only in space which is determined by the amplitude and wave number of the magnetic soliton and the nonlinear spin wave. The amplitude and wave number of spin wave have the different contribution to the width, velocity, and the amplitude of soliton solutions. Moreover, we also observe that the envelope of solution hold the shape of soliton, and the amplitude of each envelope soliton keeps invariability before and after collision which shows the elastic collision of two envelope soliton in a nonlinear spin wave background.

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